



**Assignment 2**  
Ph. D. Coursework, NAS-MUNA  
**Symmetries & Lie Algebra in Physics**  
(NWTP 702)  
Instructor: Kumar Abhinav  
Date: April 14, 2025

Due on April 28, 2025

Semester 2/2024

Total marks: 40

**Instructions**

- I. Attempt **any 4** questions out of given 5.
- II. Do not waste time by typing the answers.
- III. Do not use red ink.
- IV. The submission deadline is final.
- V. Individual marks are given in parentheses.

1. i) Why considering two independent  $SU(2)$  representations together results in addition of angular momentum? [2]  
ii) Construct all the possible states of a combined angular momentum/spin system with  $j_1 = 1/2 = j_2$ . [8]

2. i) Show that the most general parameterization of an  $SU(2)$  matrix representation is, [5]

$$D(g) = \begin{pmatrix} e^{i\xi} \cos \eta & e^{i\xi} \sin \eta \\ -e^{-i\xi} \sin \eta & e^{-i\xi} \cos \eta \end{pmatrix}.$$

- ii) Find out the matrix representations of all the generators pertaining to the above representation. [1/2+1/2+2]

3. i) Show that group multiplication is equivalent to commutators among the respective generators of a Lie group. [5]  
ii) Consider the  $su(2)$  Lie algebra,

$$[X_a, X_b] = i\epsilon_{abc}X_c, \quad \epsilon_{123} = 1.$$

Obtain the adjoint representation and verify the algebra in it. [3+2]

4. Consider the system of  $SU(2)$  generators  $J_{1,2,3}$  with  $J^2 = J_1^2 + J_2^2 + J_3^2$ .  
i) Show that  $J^2$  can have common eigenstates only with one of the three generators at once. [3]  
ii) Consider the ladder operators of  $su(2)$ ,

$$J_{\pm} = \frac{1}{\sqrt{2}} (J_1 \pm iJ_2).$$

Find out the normalization constants  $N_{\pm}^m$  in,

$$J_{\pm}|j, m\rangle = N_{\pm}^m|j, m \pm 1\rangle,$$

where, [7]

$$J^2|j, m\rangle = j(j+1)|j, m\rangle, \quad J_3|j, m\rangle = m|j, m\rangle, \quad m = -j, \dots, j, \quad j = 0, 1/2, 1, \dots$$

5. Establish the homomorphism among  $SU(2)$  and  $SO(3)$  and show that it is 2-to-1. [7+3]

**Best wishes**